Axial Vector Z' and Anomaly Cancellation

Kuo-Hsing Tsao

Ahmed Ismail, Wai-Yee Keung, KT and James Unwin

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University of Illinois at Chicago

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Less is More

One of the minimal extensions of the SM: if SM fermions are charged under a new Abelian U(1)' gauge symmetry and interacts with a new vector boson Z':

$$
\begin{aligned}
\bar{f}\not{D}f &= \bar{f}\gamma_\mu\left(\partial^\mu - igq_{f,L}\left(\frac{1-\gamma_5}{2}\right)Z'^\mu - igq_{f,R}\left(\frac{1+\gamma_5}{2}\right)Z'^\mu\right)f; \\
&= \bar{f}\gamma_\mu\left(\partial^\mu - ig(q_{f,L}+q_{f,R})\frac{1}{2}Z'^\mu - ig(q_{f,L}-q_{f,R})\frac{\gamma_5}{2}Z'^\mu\right)f
\end{aligned}
$$

However, this minimal U(1)' extension brings the anomaly into the SM if we don't introduce additional fermions.

Especially if we consider pure axial vectorial Z', i.e. $q_{f,L} = -q_{f,R}$ which has rich phenomenology interest. Here we discuss systematic methods for generating anomaly free fermions spectrum:

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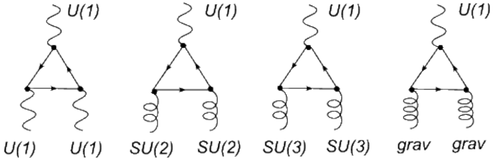
- Mirror Construction
- An Algebraic Construction
- General Algebraic Construction

The gauge anomaly occurs when the gauge current is not conserved at the triangular loop

$$< \partial_\mu j^\mu > \;\sim\; \mathcal{A}^{abc} \;=\; Tr[\gamma^5 t^a \{t^b, t^c\}] \neq 0$$

where γ^5 refers the chirality.



Other triangulars include $SU(2)^3$ and one of SU(2) or SU(3) vanish due to $Tr[t^a] = Tr[\tau^a]$ =0.

SM is Anomaly Free

$$\mathcal{A}_{WWB} := \sum_{f_L/\text{w SU(2)}} d_3[f_L]Y[f_L] - \sum_{f_R/\text{w SU(2)}} d_3[f_R]Y[f_R] = 0$$

$$\mathcal{A}_{ggB} := \sum_{f_L/\text{w SU(3)}} d_2[f_L]Y[f_L] - \sum_{f_R/\text{w SU(3)}} d_2[f_R]Y[f_R] = 0$$

$$\mathcal{A}_{GGB} := \sum_{f_L} d_2[f_L]d_3[f_L]Y[f_L] - \sum_{f_R} d_2[f_R]d_3[f_R]Y[f_R] = 0$$

$$\mathcal{A}_{BBB} := \sum_{f_L} d_2[f_L]d_3[f_L](Y[f_L])^3 - \sum_{f_R} d_2[f_R]d_3[f_R](Y[f_R])^3 = 0$$

where d_N is the dimension of the representation under SU(N), and Y is the hypercharge.

U(1)' Anomaly Free Conditions

Analogously with the SM anomaly cancellation condition, there are $\mathcal{A}_{WWZ'}$, $\mathcal{A}_{ggZ'}$, $\mathcal{A}_{GGZ'}$, and $\mathcal{A}_{Z'Z'Z'}$ and two additional cancellation needed from U(1)-U(1)' mixing:

$$\begin{aligned}
\mathcal{A}_{Z'Z'B} &:= \sum_{f_L} d_2[f_L]d_3[f_L]Y[f_L](z[f_L])^2 - \sum_{f_R} d_2[f_R]d_3[f_R]Y[f_R](z[f_R])^2 \\
&= 0 \\
\mathcal{A}_{BBZ'} &:= \sum_{f_L} d_2[f_L]d_3[f_L]z[f_L](Y[f_L])^2 - \sum_{f_R} d_2[f_R]d_3[f_R]z[f_R](Y[f_R])^2 \\
&= 0
\end{aligned}$$

Exotics and Anomaly Free

In the case of a pure Axial vector coupling to both the SM and DM:

$$z_q^{(i)} := z_Q^{(i)} = -z_u^{(i)} = -z_d^{(i)};\ z_l^{(i)} := z_L^{(i)} = -z_e^{(i)};\ z_{\mathrm{DM}} := z_{\chi L} = -z_{\chi R}$$

Field Name	$\mathrm{U}(1)_Y$	$\mathrm{SU}(2)_L$	SU(3)	Notation
$Q_L^i,\ Q'_{L,R}$	$1/3$	2	3	$(3,2)_{(1/3,\ z)}$
$u_R^i,\ u'_{L,R}$	$4/3$	1	3	$(1,2)_{(4/3,\ z)}$
$d_R^i,\ d'_{L,R}$	$-2/3$	1	3	$(3,1)_{(-2/3,\ z)}$
$L_L^i,\ L'_{L,R}$	-1	2	1	$(1,2)_{(-1,\ z)}$
$e_R^i,\ e'_{L,R}$	-2	1	1	$(1,1)_{(-2,\ z)}$
$S_{L,R},\ \nu_R,\ \chi_{L,R}$	0	1	1	$(1,1)_{(0,\ z)}$

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- e.x. Q_L carries $\mathrm{U}(1)'$ charge z_q (i.e. $(3,2)_{1/3,z_q}$), one might add either a LH exotic in the representation $(3,2)_{-1/3,-z_q}$ or a RH exotic in $(3,2)_{1/3,z_q}$

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- Cancellation happens state-by-state and generation-by-generation

An Algebraic Construction

Assume $z_{\mathrm{SM}} := z_q = z_l$ and introduce vector-like exotics

$Q'_L,\ Q'_R,\ u'_L,\ u'_R,\ d'_L,\ d'_R,\ l'_L,\ l'_R,\ e'_L,\ e'_R,$

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Anomaly Free Conditions

Straightforwardly, anomaly cancellation is solved by a general and unique set of equations:

$$z_{Q'_R} = z_{Q'_L} + 2z_{\mathrm{SM}}, \quad z_{u'_R} = 7z_{\mathrm{SM}} + z_{u'_L},$$

$$z_{d'_R} = z_{d'_L} + z_{\mathrm{SM}}, \quad z_{L'_R} = z_{L'_L} + 6z_{\mathrm{SM}}, \quad z_{d'_L} \neq 2z_{Q'_L} + \frac{101}{2} z_{\mathrm{SM}} + 14 z_{u'_L},$$

$$z_{e'_L} = \frac{1}{3}(z_{d'_L} + 6z_{L'_L} - 2z_{Q'_L} - 28z_{\mathrm{SM}} - 14z_{u'_L}),$$

$$z_{e'_R} = \frac{1}{3}(z_{d'_L} + 6z_{L'_L} - 2z_{Q'_L} - 37z_{\mathrm{SM}} - 14z_{u'_L}),$$

$$z_{L'_L} = \frac{1}{\Omega}\Big(-8z_{d'_L}^2 - 4z_{d'_L}z_{Q'_L} - 32z_{Q'_L}^2 - 74z_{d'_L}z_{\mathrm{SM}} + 58z_{Q'_L}z_{\mathrm{SM}} - 404z_{\mathrm{SM}}^2 - 28z_{d'_L}z_{u'_L} + 56z_{Q'_L}z_{u'_L} + 469z_{\mathrm{SM}}z_{u'_L} + 133z_{u'_L}^2 \Big),$$

where $\Omega = 606z_{\mathrm{SM}} + 168z_{u'_L} - 12z_{d'_L} + 24z_{Q'_L}$

If $z_{\rm SM} = z_{Q'_L} = 1$ and $z_{u'_L} = -z_{Q'_R} = -3$:

$$z_{u'_R} = 4, \quad z_{d'_L} = 3, \quad z_{d'_R} = 4, \quad z_{L'_L} = -9,$$

$$z_{L'_R} = -3, \quad z_{e'_L} = -13, \quad z_{e'_R} = -16.$$

A Selection of Anomaly Free Axial Vector Models

	n_G	Universal?	Lepto-phobic/philic?
♯1. /wo DM Model	3	✓	✗
♯2. /w DM Model	3	✓	✗
♯3. L-phobic Model	3	✓	Leptophobic
♯4. L-phliic Model	3	✓	Leptophilic
♯5. 1G-Model	1	N/A	✗
♯6. t-b-Model	1	N/A	Leptophobic

A Selection of Anomaly Free Axial Vector Models

Field	♯1	♯2	♯3	♯4	♯5	♯6
$z[Q_L]$	1	1	1	0	1	1
$z[u_R]$	-1	-1	-1	0	-1	-1
$z[d_R]$	-1	-1	-1	0	-1	-1
$z[L_L]$	1	1	0	1	1	0
$z[e_R]$	-1	-1	0	-1	-1	0

A Selection of Anomaly Free Axial Vector Models

Field	♯1	♯2	♯3	♯4	♯5	♯6
$z[\chi_L]$	-	9	9	-9/4	1	1
$z[\chi_R]$	-	-9	-9	9/4	-1	-1
$z[Q'_L]$	1	1	1	-	0	0
$z[Q'_R]$	3	-1	0	-	1	1
$z[u'_L]$	-3	-2	-2	-2	-1	-1
$z[u'_R]$	4	3	-1	5/2	0	0
$z[d'_L]$	3	-6	-2	2	-1	-1
$z[d'_R]$	4	5	11	-5/2	0	0
$z[L'_L]$	-9	-82/3	-49/12	-157/48	0	0
$z[L'_R]$	-3	-28/3	95/12	-13/48	1	0
$z[e'_L]$	-13	-100/3	103/6	-85/24	-1	0
$z[e'_R]$	-16	-127/3	67/6	-121/24	0	0

A Selection of Anomaly Free Axial Vector Models

Field	$\sharp 1$	$\sharp 2$	$\sharp 3$	$\sharp 4$	$\sharp 5$	$\sharp 6$
$z[\nu_R]$	-	-	-	-	1	1
$N[\nu_R]$	-	-	-	-	2	2

SM Fermions

For axial vector couplings $z[\bar{Q}_L u_R] = 2z_0$ and $z[\bar{Q}_L d_R] = 2z_0$, the gauge invariant mass operator $H^\dagger \bar{Q}_L u_R$ requires that
$z[H^\dagger] = -2z_0$.
However, $H\bar{Q}_L d_R$ breaks down U(1)' gauge invariance. There are two ways out:

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- Type II Two Higgs Doublet Model
- EFT higher dimension operators $\frac{1}{\Lambda} S H^\dagger Q_L \overline{u}_R$. S is a SM singlet and gets vev $\langle S \rangle \equiv v'$ to break U(1)' (Froggatt-Nielson).

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- $m_{Z'} \simeq g'v'$; $m_S \simeq \lambda_S v'$ (λ_S is the S quadratic coupling);

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- Unitarity tells us $m_f,\ m_\chi \lesssim \frac{m_{Z'}}{g'} \simeq v'$ (F. Kahlhoefer, *et al.* JHEP **1602** (2016) 016)
- The heavy DM and top give the constraint: $m_{Z'} \gtrsim 3.5\ \mathrm{TeV} \left(\frac{g'}{\sqrt{4\pi}}\right)\left(\frac{m_{\mathrm{DM}}}{1\ \mathrm{TeV}}\right)$

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- UV completeness also requires the introduction of new vector like fermions.
- $m_{Z'} \sim g'v' \sim M$ and the exotics ($M \sim y'v'$):

$$M \lesssim v' \simeq \frac{m_{Z'}}{g'(m_{Z'})}.$$

Model $\sharp 1$

$z[H] = 2$, $z[S_1] = 1$ and $z[S_4] = 4$

$$\mathcal{L}_{\rm SMY} \supset y_u^i H \bar{Q}_L u_R + \frac{y_d^i}{\Lambda} S_4 H^\dagger \bar{Q}_L d_R + \frac{y_l^i}{\Lambda} S_4 H^\dagger \bar{L}_L e_R$$

$$\mathcal{L}_{\rm Ex} \supset y_{Q'} S_1^2 Q'_L \bar{Q}'_R + \frac{y_{u'}}{\Lambda^2} S_4^2 S_1^\dagger u'_L \bar{u}'_R + y_{d'} S_1 d'_L \bar{d}'_R + \frac{y_{L'}}{\Lambda^2} S_4 S_1{}^2 L'_L \bar{L}'_R + \frac{y_{e'}}{\Lambda} S_4 S_1^\dagger e'_L \bar{e}'_R$$

$$L_{\rm UV} \supset y_\psi H^\dagger \bar{L}_L \psi_L + y'_\psi S_4 \bar{\psi}_L e_R + m_\psi \bar{\psi}_L \psi_R$$

where $\Lambda = \frac{m_\psi}{y_\psi y'_\psi}$ and ψ_L, ψ_R in the representation $(1,1)_{-2,3}$

Mass Generation For Mirror Construction

In order to have above EW massive mirror exotics, additional U(1)' neutral charged exotics fermions are introduced:

$$\mathcal{L}_{Mir} \supset S\bar{Q}'_L Q'_R + S\bar{u}'_L u'_R + S\bar{d}'_L d'_R + S\bar{L}'_L L'_R + S\bar{e}'_L e_R \ .$$

where $z[Q'_L] = z[u'_R] = z[d'_R] = z[L'_L] = z[e'_R] = 0$ and $z[S] = -1$

The Non-Perturbative Limit

U(1)$'$ coupling strength $\alpha' \equiv g'^2/4\pi$ runs with the energy scale Q is given by

$$\frac{d\alpha'^{-1}}{d\ln Q} = -\frac{b}{2\pi} \qquad \text{with} \qquad b = \sum_f \frac{2}{3} z_f^2 + \sum_s \frac{1}{3} z_s^2$$

If the new fermions enter at the scale M, the running of g' to some UV scale Λ is described by

$$\alpha'^{-1}(\Lambda) = \alpha'^{-1}(m_{Z'}) - \int_{m_{Z'}}^{M} \frac{b_{Z'}}{2\pi}\,\mathrm{d}\ln Q - \int_M^{\Lambda} \frac{b_{Z'} + b_M}{2\pi}\,\mathrm{d}\ln Q$$

where $b_{Z'}$ the sum is over the SM states and DM.

At the scale $\Lambda_{\not{P}}$, the coupling becomes non-perturbative $\alpha'(\Lambda_{\not{P}}) \sim 1$ (Landau Pole)

$$\Lambda_{\not{P}} = M \exp\left[\frac{1}{b_M + b_{Z'}}\left(b_{Z'}\log\left[\frac{m_{Z'}}{M}\right] + \frac{2\pi}{\alpha'(m_{Z'})} - \frac{2\pi}{\alpha'(\Lambda_{\not{P}})}\right)\right]$$

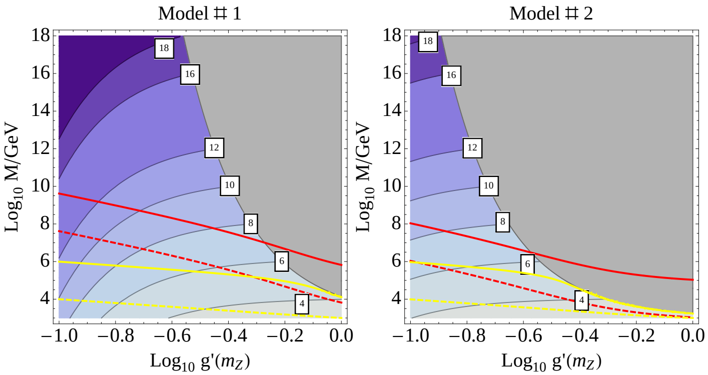
The Non-Renormalizable Limit

The scale limit $\Lambda_{\not{R}}$ for an anomaly EFT theory maintaining renormalizable without introducing exotics to cancel anomaly at scale M is: (Preskill 1991)

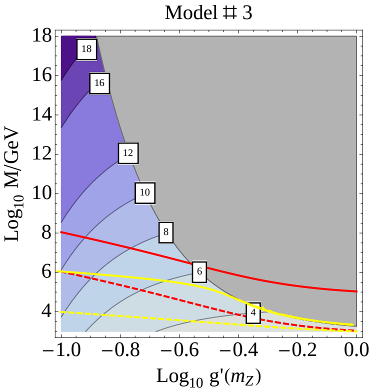
$$M < m_{Z'} \left(\frac{64\pi^3}{|g_{\not{R}}^3 \mathcal{A}_{Z'Z'Z'}|} \right) \equiv \Lambda_{\not{R}}$$

where $g_{\not{R}} \equiv g(\Lambda_{\not{R}})$ and $\mathcal{A}_{Z'Z'Z'} = \mathrm{Tr}[z^3]$ is the $\mathrm{U}(1)'^3$ anomaly coefficient calculated in the EFT below the scale of the exotics M.

Breakdown of Low Energy Theories



PURPLE CONTOURS: g' runs non-perturbative. GREY REGION: $\Lambda_{\not P} < M$. RED CURVES: $\Lambda_{\not R}$ for $m_{Z'} \sim 1$ TeV(DASHED), and 100 TeV(SOLID). YELLOW CURVES: Unitarity for $m_{Z'} \sim 1$ TeV(DASHED), and 100 TeV(SOLID).


Model ♯ 3
Model ♯ 4
18
16
14
12
10
8
6
4
Log_{10} M/GeV
−1.0 −0.8 −0.6 −0.4 −0.2 0.0
$\mathrm{Log}_{10}\ g'(m_Z)$

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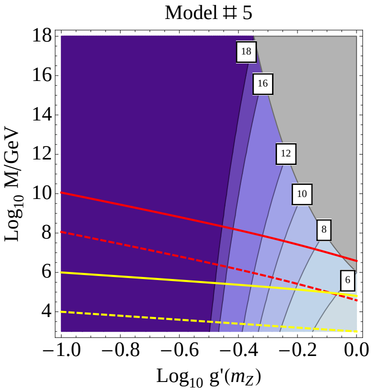

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- The additional states required for anomaly cancellation with potentially important constraints have been highlighted.
- The study of phenomenological issues for the t-b-philic case is coming soon (Model ♯6). arXiv 1701.xxxxx

Thank you!!!